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Geometric Reasoning

GCSE Further Maths

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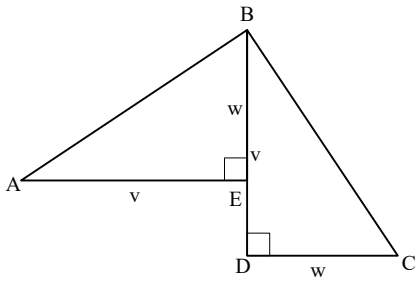
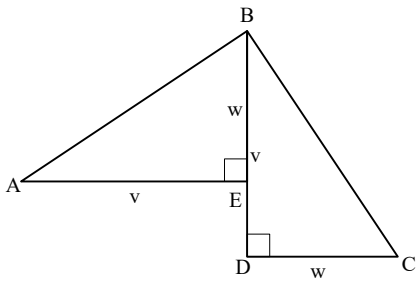
GEOMETRIC REASONING

You **must** show all your working and, where necessary, give reasons for each of your statements.

Q1 Show that the two triangles below are congruent.

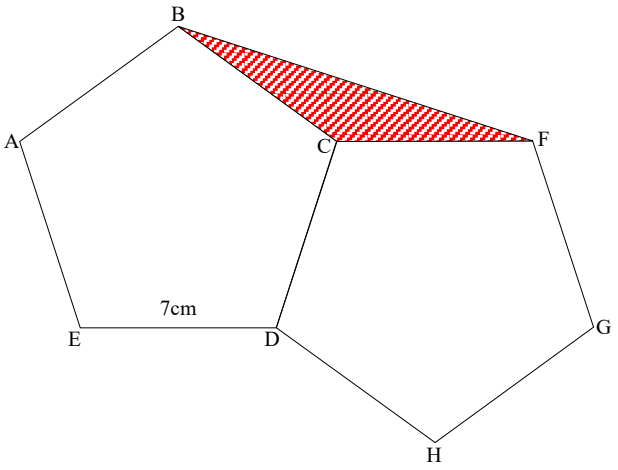


Q2 Prove that angle ABC is a right angle. (Include the proof of congruency in this answer)

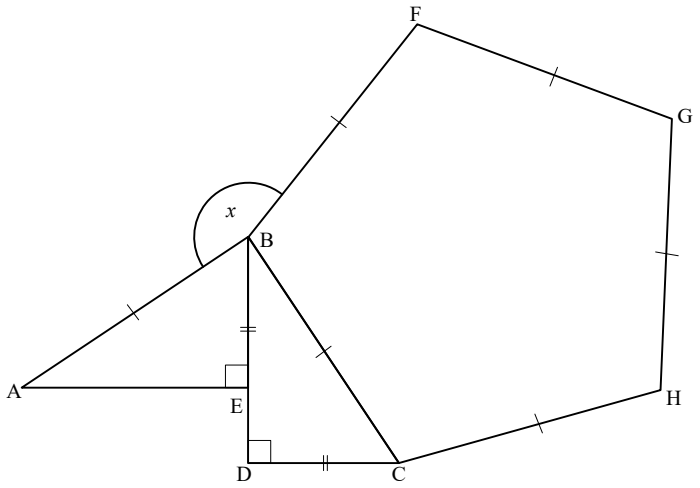


Q3 ABCDE and CDHGF are two regular pentagons.

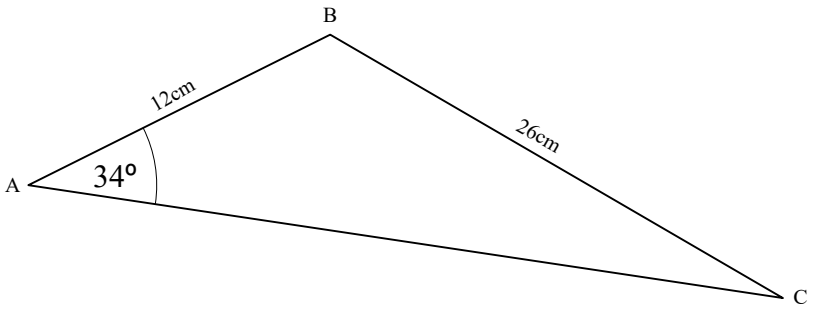
Find the area of BCF.



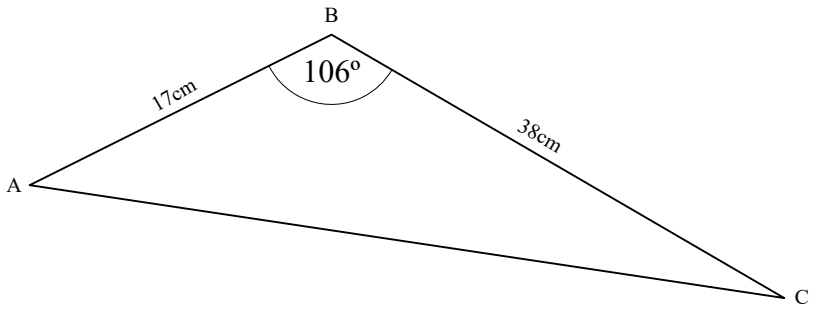
Q4 Prove that angle x is $162°$



Q5 Find the size of angle BCA.



Q6 ABC is a triangle.



a Find the length of AC.

b Find the area of ABC.

Q7 A triangle has ides 8.4cm, 9.2cm and 6.7cm.

Calculate the area of the triangle.

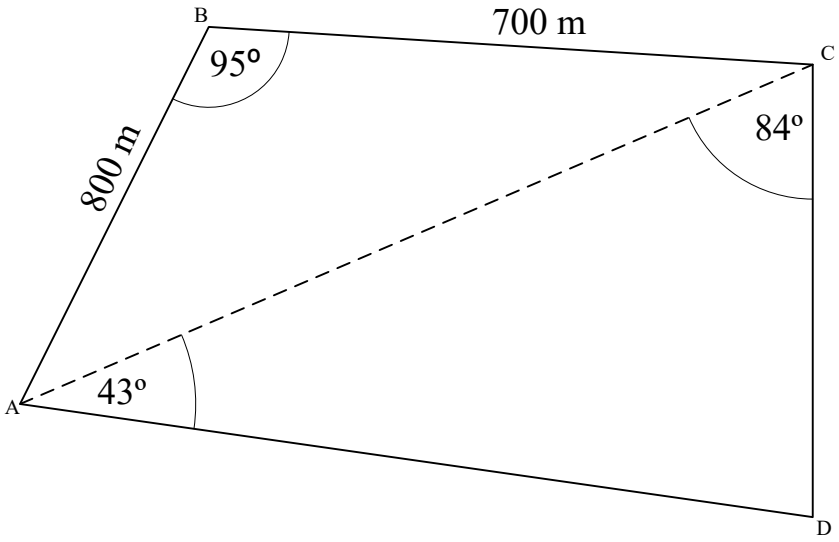
Q8 From her house, Heidi walks 5km on a bearing of 080°. She then walks 7.8km on a bearing of 126°.

How far and in what direction must she walk to return home?

Q9 Farmer Declan wants to buy some land.

Land costs £9,660 per hectare.

1 hectare = 10,000 m^2

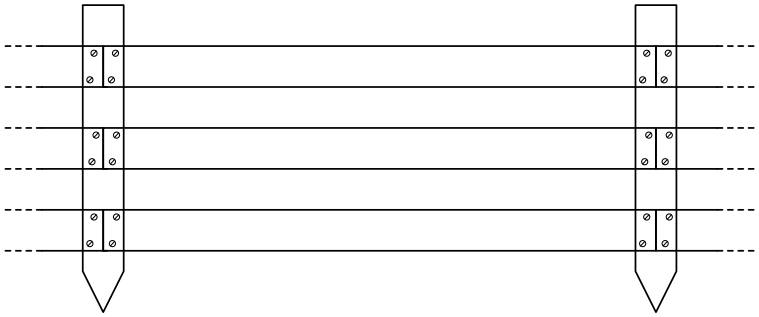


How much will the land cost him?

Q10 Declan wants to split his land into two fields by building a fence along the line AC.

Upright posts cost £20 each and the crossbars for the fence cost £4.80 each. He wants the fence to have three crossbars all the way along its length. The length of each crossbar is 1.8m.

The ratio of sand to cement to mix the concrete is 3:1. Each upright needs 3kg of concrete to hold it firmly in position. Both the sand and the cement are sold in 25kg bags. Sand costs £6 per bag and cement costs £8.95 per bag. Screws cost 12p each.



How much does it cost to build the fence?

Q11 Cinnamon Perfume is sold in bottles in the shape of a square based pyramid. The vertex is directly above the centre of the base. The base has edges of 10cm whilst the sloping edges from the base to the vertex are 13cm long.

The bottle is 80% full of perfume. Perfume retails at £7.80 for 10ml.

i Find the angle between the slanting edge and the base of the pyramid.

ii Find the angle between a side and the base.

iii For how much should a bottle of Cinnamon Perfume be sold?

Q12 Point A(-4,12) and point B(6,-3) are co-ordinates at each end of line segment K.

Line segment L is the perpendicular bisector of K.

The length of K and L are equal.

Points C and D are the end points of L.

Find the co-ordinates of points C and D.

Q13 The three points A(-7,12), B(x,17) and C(8,23) are collinear (are on the same line).

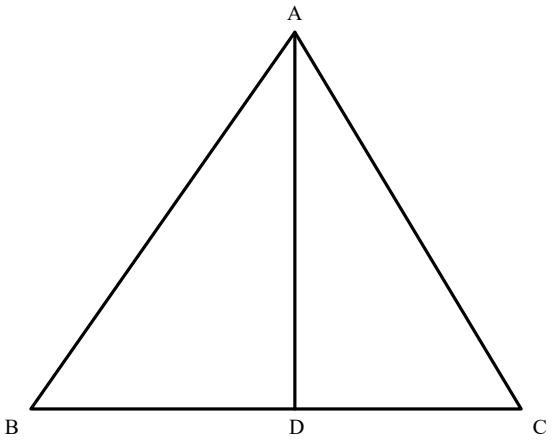
Find the value of x and hence the ratio of AB:BC.

Q14 The line $5x - 3y + 15 = 0$ cuts the x-axis at A and the y-axis at B. Find the area of AOB where O is the origin.

Q15 ABC is a triangle.

The perpendicular at A meets BC at D.

$BC = \left(6 + 2\sqrt{7}\right)$ cm,



Do NOT scale

Area of triangle $ABC = (13 + 3\sqrt{7})$ cm.

Work out the length of line BC giving your answer in the form $a + b\sqrt{c}$ where a, b and c are integers.

Q16 For each of the following questions, find θ where $0° \leq \theta \leq 360°$

a $\sin\theta = 0.5$

b $\cos\theta = -0.5$

c $\cos\theta = \frac{\sqrt{3}}{2}$

d $\tan\theta = -\sqrt{2}$

e $\tan\theta = 2$

f $\sin\theta = -\frac{\sqrt{3}}{2}$

Q17 a Factorise $3x^2 - x - 2$

b Solve $3x^2 - x - 2 = 0$

c Use your results to solve the equations for $0° \leq \theta \leq 360°$:

i $3cos^2\theta - cos\theta - 2 = 0$

ii $3sin^2\theta - sin\theta - 2 = 0$

iii $3tan^2\theta - \tan\theta - 2 = 0$

Q18 Write down the equations of these circles.

a centre (4,9) radius 12

b centre (6,1) radius 5

c centre (7,5) radius 9

d centre (9,12) radius 4

e centre (-5, $\sqrt{3}$) radius $2\sqrt{3}$

f centre (-9, $1\frac{4}{5}$) radius $5\sqrt{7}$

g centre (-8, 2+$3\sqrt{2}$) radius $7\sqrt{11}$

Q19 A circle with centre at (-2,5) passes through the point (12, 9). Work out the equation of the circle.

Q20 A circle with centre (-1,1) runs through the point (2,5). Find the equation of the circle.

Q21 A circle with centre (-4, 2) runs through the point (5,8). Find the equation of the circle.

Q22 Sketch the circle $(x-2)^2+(y+3)^2=25$ which goes through the points A(2,2), B(6,0) and C(-2,0). Lines J, K and L are tangents of A, B and C respectively.

i Find the equations of each of the tangents J, K and L.

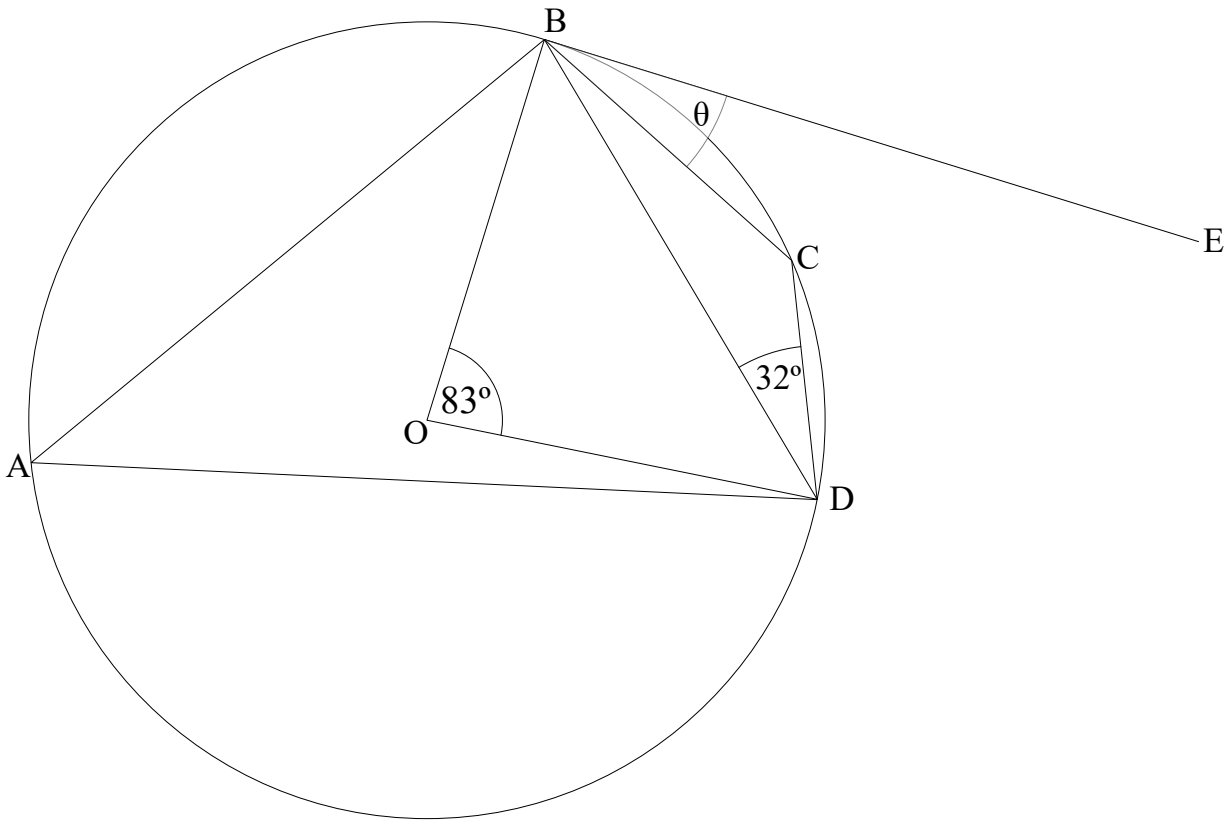
ii Find the point where tangents J and K intercept.

iii Find the point where tangents K and L intercept.

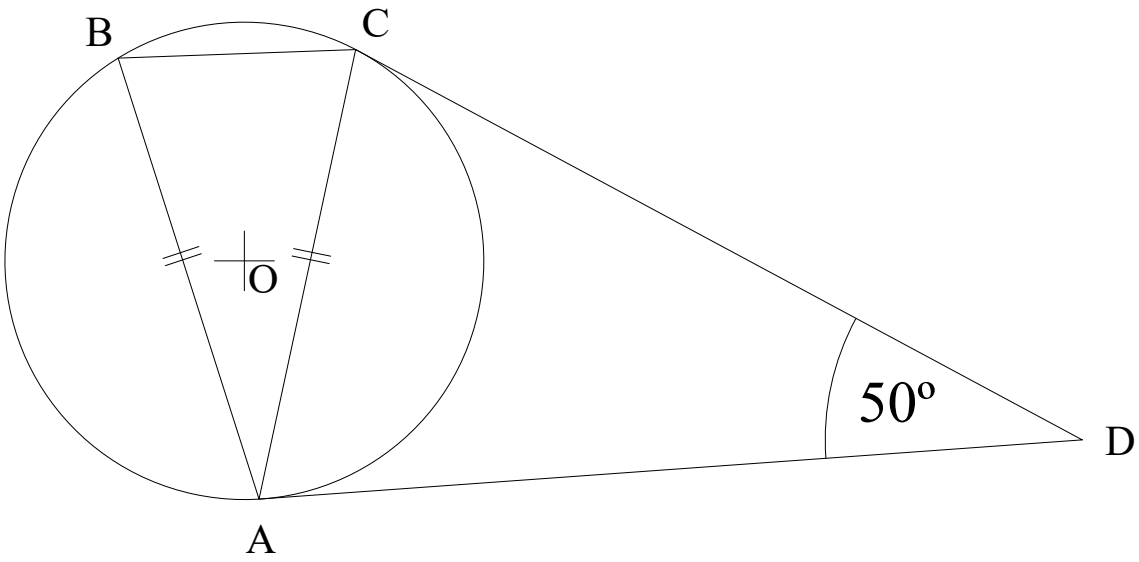
Q23 Below is a circle centred at O. BE is a tangent to the circle.

Angle BDC is 32° and angle BOD is 83°.

Find angle CBE. You **MUST show reasons** for each stage of the calculation.

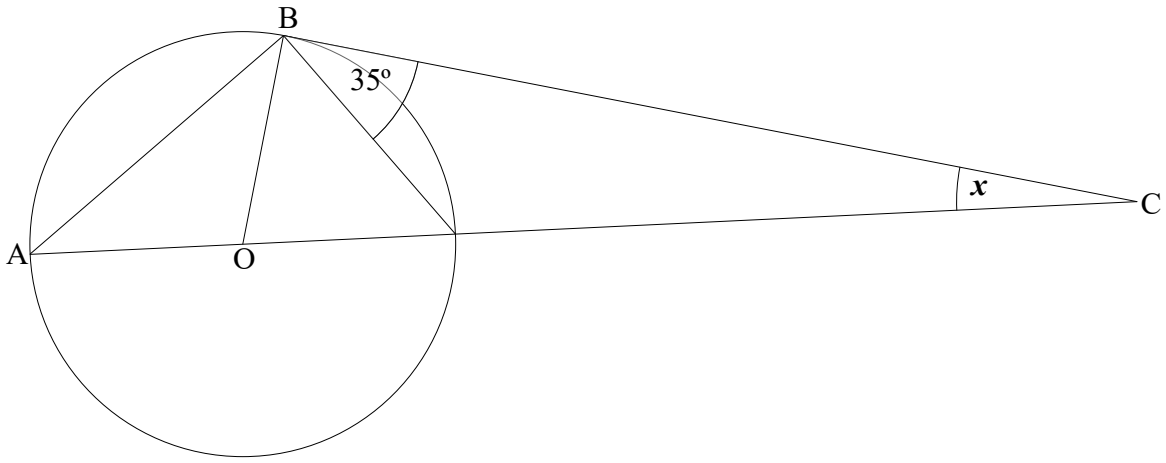


Q24 Find the size of each of the angles in triangle ABC.

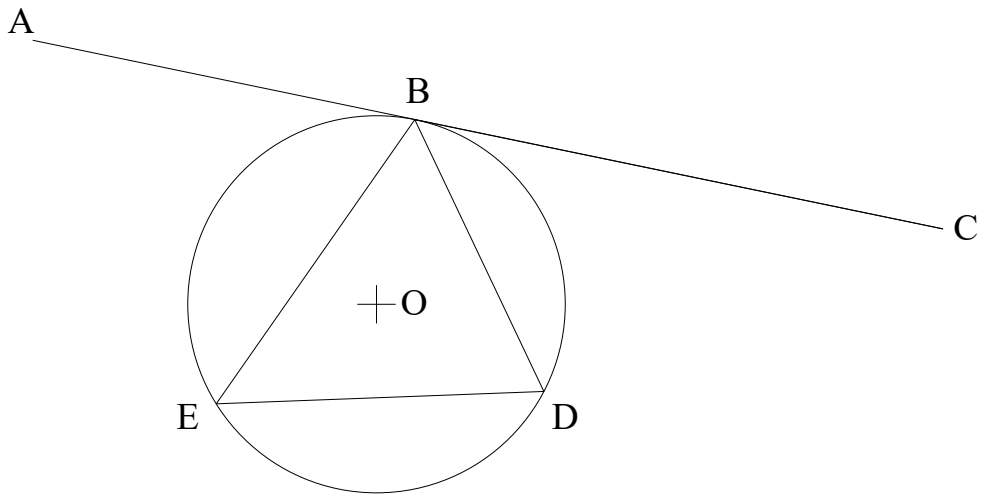


Q26 Find the size of angle x.

You **must** give reasons for each statement you make.



Q27 Prove the alternate segment theorem.



Q28 Prove that the angle subtended by a diameter is equal to 90°